$$U(P) = \int_P^{rif} \vec{F}_{el} \cdot \vec{ds} \qquad V(P) = \frac{U(P)}{q} = \int_P^{rif} \vec{E} \cdot \vec{ds}$$

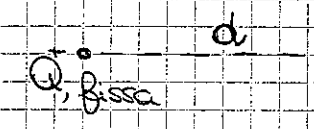
$$F_x = -\frac{\partial U_{pot}}{\partial x} \quad \ldots \Rightarrow \vec{F} = -\vec{\nabla} U_{pot}$$

$\left\{\begin{array}{l} F \perp \text{alle sup. equipotenziali} \\ F \text{ diretta nel verso decrescente dell'energia potenziale} \end{array}\right.$

$$\boxed{\vec{F} = -\vec{\nabla} U_{pot} \Rightarrow \vec{E} = -\vec{\nabla} V}$$

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Esercizio



velocità quando si trova a $2d$

Si conserva l'en. meccanica.

$$E_{cin}^i + E_{pot}^i = E_{cin}^f + E_{pot}^f$$

$$qV(r=d) = \frac{1}{2} m v^2 + qV(r=2d)$$

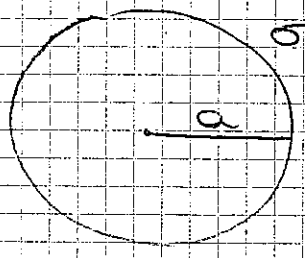
$$\frac{KQq}{d} = \frac{1}{2} m v^2 + \frac{KQq}{2d}$$

$$v = \sqrt{\frac{2q}{m}\left(\frac{KQ}{d} - \frac{KQ}{2d}\right)} = \sqrt{\frac{KQq}{m}}$$

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Calcolo potenziale



$\vec{E}(r)$ con $r<a$ $\quad \vec{E}(r) = 0$

$\vec{E}(r)$ con $r>a$ $\quad \vec{E}(r) = \frac{K 4\pi a^2 \sigma}{r^2} \hat{r}$

$$V(P) = \int_P^{rif} \vec{E} \cdot \vec{ds} =$$

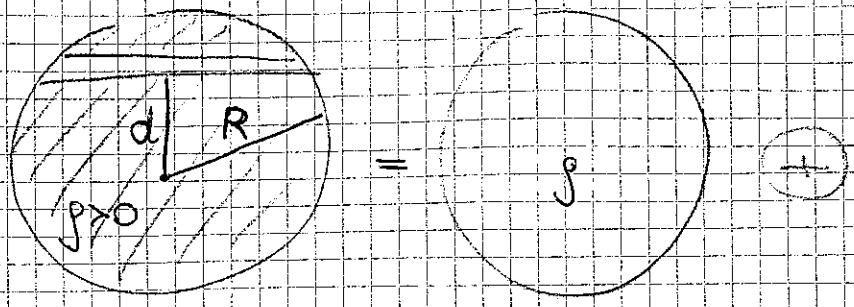
$$= \int_P^{rif} \frac{K 4\pi a^2 \sigma}{r^2} \hat{r} \cdot \vec{ds} =$$

$$= \int_P^{rif} \frac{4K\sigma\pi a^2}{r^2} dr$$

$r > a \quad V(P) = \frac{4K\sigma\pi a^2}{r}$

$r < a$

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Campo elettrico sfera

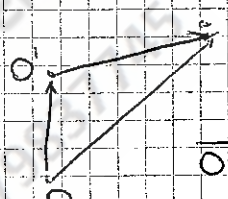
$$\vec{E}_{①} = \frac{\rho}{3\varepsilon_0}\,\vec{OP} \qquad \vec{E}_{②} = -\frac{\rho}{2\varepsilon_0}\,\vec{O'P}$$

$$E_②(r)\,2\pi r h = -\frac{\rho \pi r^2 h}{\varepsilon_0}$$

$$Q_{int}(r) = -\int \rho\, dV = -\rho \int 2\pi r h\, dr = -2\rho\pi h \int r\, dr =$$

$$= -\rho\pi h r^2$$

$$\vec{E}_②(r) = -\frac{\rho}{2\varepsilon_0}\,\vec{r}$$



$$\vec{OP} = \vec{OO'} + \vec{O'P}$$

$$\vec{E}_① + \vec{E}_② = \frac{\rho}{3\varepsilon_0}\,\vec{OP} - \frac{\rho}{2\varepsilon_0}\,\vec{O'P} =$$

$$= \frac{\rho}{3\varepsilon_0}\,\vec{OP} - \frac{\rho}{2\varepsilon_0}\left(\vec{OP} - \vec{OO'}\right) =$$

$$= \frac{\rho}{3\varepsilon_0}\left(\vec{d} + \vec{x}\right) - \frac{\rho}{2\varepsilon_0}\,\vec{x} = \frac{\rho}{3\varepsilon_0}\,\vec{d} + \frac{\rho}{\varepsilon_0}\,\vec{x}\left(\frac{1}{3} - \frac{1}{2}\right) =$$

$$= \frac{\rho}{3\varepsilon_0}\,\vec{d} - \frac{\rho}{6\varepsilon_0}\,\vec{x} = \frac{\rho}{6\varepsilon_0}\left(2\vec{d} - \vec{x}\right)$$

$$\max \ell = \frac{\rho}{6\varepsilon_0}(2d - \ell)$$

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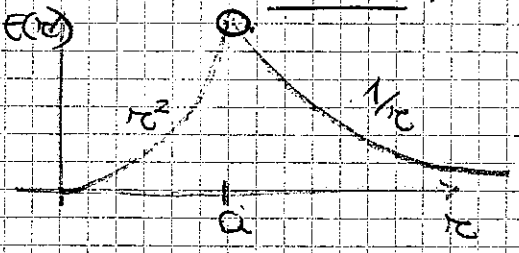
Th. di Gauss

$$① E(r) \cdot 2\pi r h = \frac{2}{3}\frac{\pi \alpha r^3 h}{\varepsilon_0}$$

$$E(r) = \frac{\alpha}{3\varepsilon_0} r^2 \qquad r < a$$

$$② E(r) 2\pi r h = \frac{2}{3}\frac{\pi \alpha h a^3}{\varepsilon_0}$$

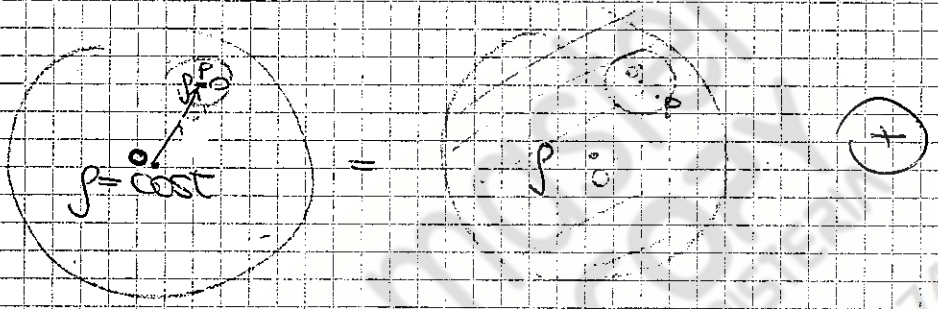
$$E(r) = \frac{\alpha a^3}{3\varepsilon_0} \cdot \frac{1}{r} \qquad r > a$$



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Campo nel buco sferico



Sup. di Gauss: sfera concentrica a r variabile

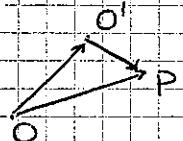
$$\Phi = \int \vec{E} \cdot d\vec{A} = \int E(r)\hat{r} \cdot dA \cdot \hat{r} = \int E(r)\, dA = E(r) 4\pi r^2$$

Con il th. di Gauss calcolo

$$E_① = \frac{\rho}{3\varepsilon_0} \cdot \overline{OP} \qquad E_② = -\frac{\rho}{3\varepsilon_0} \overline{O'P}$$

Per calcolare E_{D_q} applico il pr. di sovrapposizione:

$$\vec{E}_{D_q} = \vec{E}_① + \vec{E}_② = \frac{\rho}{3\varepsilon_0}\overline{OP} - \frac{\rho}{3\varepsilon_0}\overline{O'P} = \frac{\rho}{3\varepsilon_0}\left(\overline{OP} - \overline{O'P}\right)$$



$$\overline{OP} = \overline{OO'} + \overline{O'P} \Rightarrow \overline{OP} - \overline{O'P} = \overline{OO'}$$

$$\vec{E}_{D_q} = \frac{\rho}{3\varepsilon_0} \cdot \overline{OO'} \Rightarrow$$ non dipende dal punto

$\Downarrow$

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② ρ uniforme

$$Q_{int}(x) = \begin{cases} |x| > d/2 \\ |x| < d/2 \end{cases}$$

$$Q_{int} = \rho d \cdot A_{base}$$

$$Q_{int} = \rho \cdot 2x \, A_{base}$$

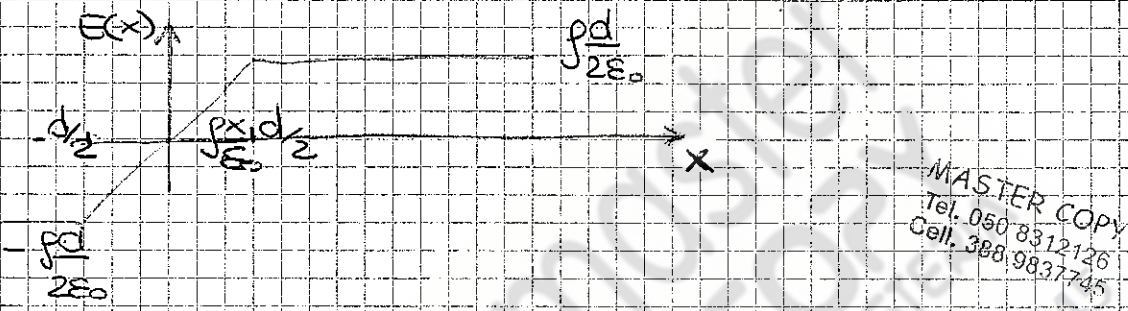
Per la distribuzione piana vale:

$$\Phi = 2E(x) \cdot A_{base}$$

$$E(x) = \begin{cases} \frac{\rho d}{2\varepsilon_0} \\ \frac{\rho 2x}{2\varepsilon_0} = \frac{\rho x}{\varepsilon_0} \end{cases}$$

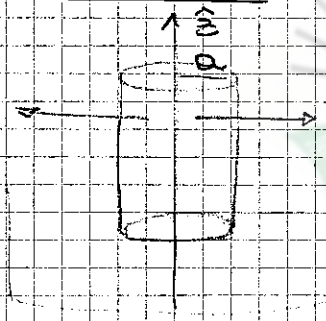
$$|x| > d/2 \quad \begin{cases} \frac{\rho d}{2\varepsilon_0} & x > d/2 \\ -\frac{\rho d}{2\varepsilon_0} & x < -d/2 \end{cases}$$

$$|x| < d/2$$



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$$\begin{cases} \rho = \alpha r & r < a \\ = 0 & r > a \end{cases}$$

$L >> a$

$$\vec{E} = E(r) \cdot \hat{r}$$

Superficie gaussiana: cilindro coassiale

$$\Phi = \int \vec{E} \cdot d\vec{A} = \int E(r) \hat{r} \cdot (dA_{lat} \cdot \hat{r}) + 2\int E(r) \hat{r} \cdot dA_{base} \hat{z} =$$

$$= \int E(r) \, dA_{lat} = E(r) A_{lat} =$$

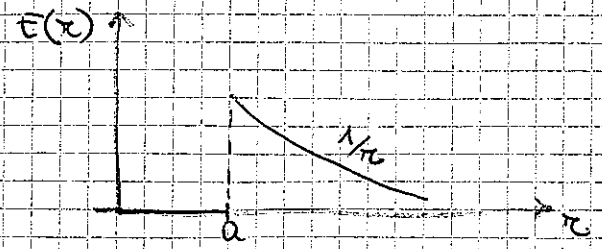
$$= E(r) \cdot 2\pi r h$$

$$Q_{int}(r) = \int dq = \int \rho dV =$$

$$= \int \alpha r \cdot 2\pi r h \, dr = \int 2\alpha\pi h \cdot r^2 dr = 2\alpha\pi h \left[\frac{r^3}{3}\right]$$

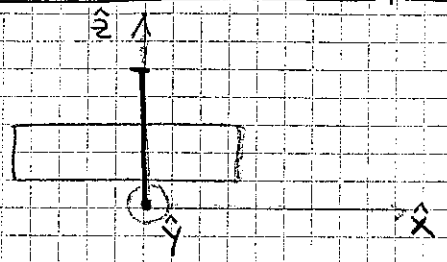
$$Q_{int} = \begin{cases} \frac{2}{3}\pi\alpha h r^3 & r < a \\ \frac{2}{3}\pi\alpha h a^3 & r > a \end{cases}$$

$$E(r) = \frac{2\pi a h \sigma}{\varepsilon_0 2\pi r h} = \frac{\sigma a}{\varepsilon_0}\frac{1}{r} \qquad r > a$$



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Distribuzione piana



$$\bar{E}(x,y,z) = \bar{E}(x,y,z)\hat{x} = E(x)\hat{x}$$

$$\bar{E}(x) = -\bar{E}(-x)$$

Superficie di Gauss: cilindro con sup di base // al piano

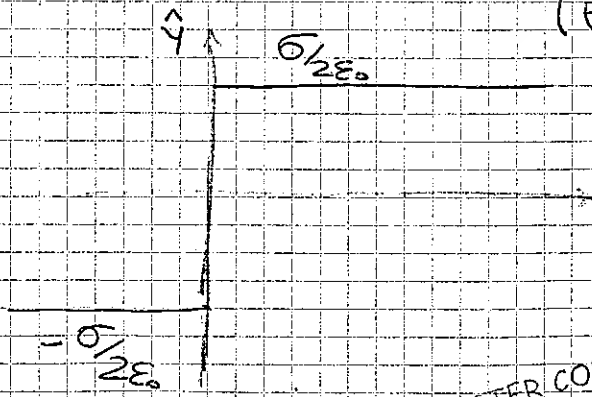
$$\Phi = \int \bar{E}\cdot d\bar{A} = \int_{2\,sup.\,base} \bar{E}\cdot d\bar{A} + \int_{sup.\,lat.} \bar{E}\cdot d\bar{A} = \int_{2\,sup.\,lat.} \bar{E}\cdot d\bar{A} = \int_{2\,sup} E(x)\hat{x}\cdot\hat{x}\,dA =$$

$$= \int_{x>0} E(x)\,dA + \int_{x<0} E(-x)\,\hat{x}\cdot(-\hat{x})\,dA = E(x)A_{base} + E(x)A_{base} =$$

$$= 2E(x)A_{base}$$

$$dQ_{int} = \sigma\, dA_{base} \Rightarrow 2E(x)A_{base} = \frac{\sigma A_{base}}{\varepsilon_0}$$

$$\begin{cases} E(x) = \dfrac{\sigma}{2\varepsilon_0} & x>0 \\ E(x) = -\dfrac{\sigma}{2\varepsilon_0} & x<0 \end{cases}$$



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